Probabilistic Forecasting Based on Structural Health Monitoring

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OBJECTIVE: Develop a Health Usage Monitoring System (HUMS) to monitor fatigue damage

OUTLINE:

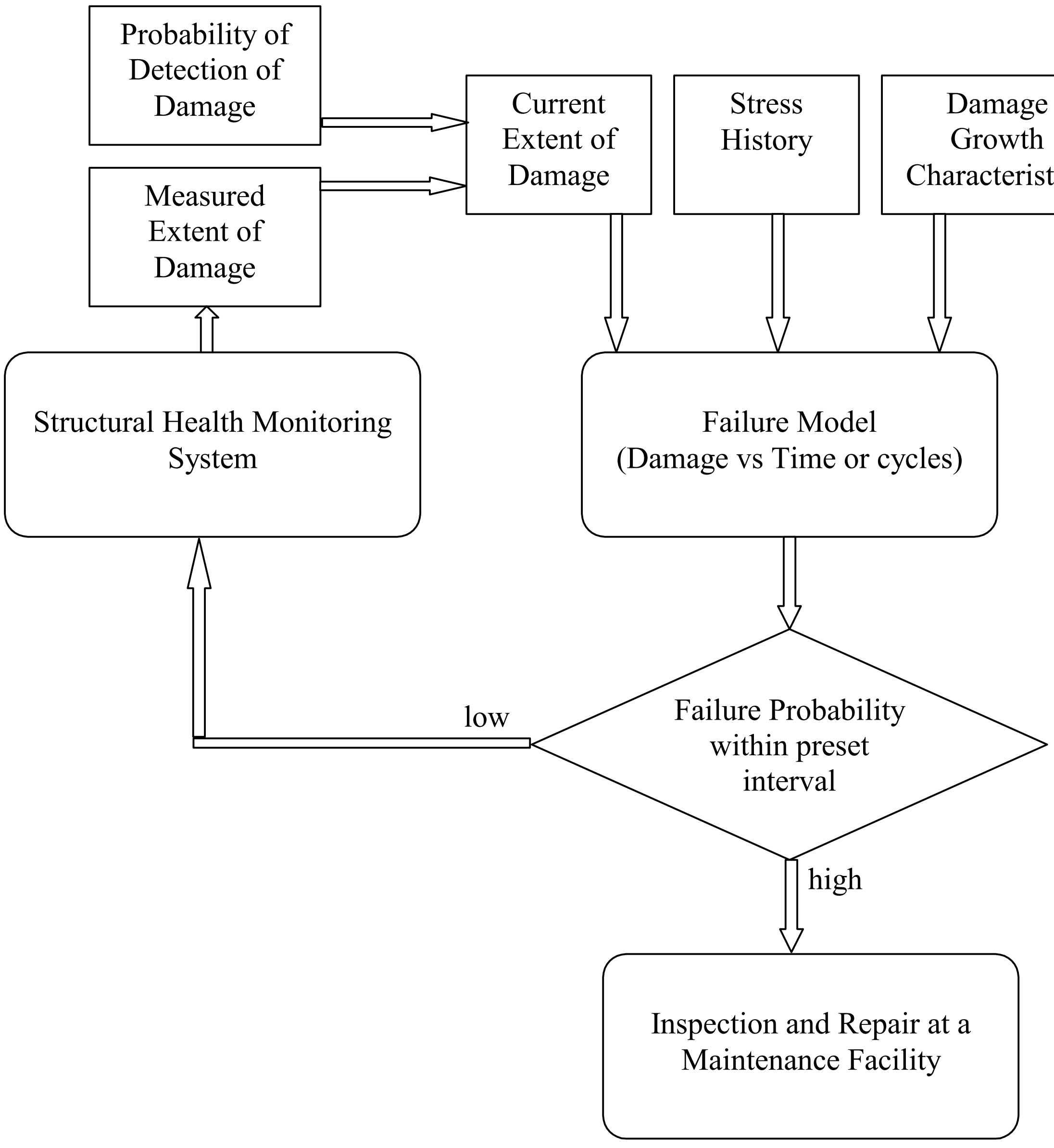
- Grand Plan
- Stage 1: Development of fatigue damage leading to macrocrack
- Stage 2: Growth of macrocrack
- Stage 3: Probability of an undetected crack with crack length $a > a_{cr}$

Rotorcraft Structures/HUMS Project Review, December 7 - 9, 2004

Grand Plan

- permanently installed microsensors
- continuous monitoring in real time with known POD
- wireless transmission to central station
- instantaneous interpretation of sensor data
- detection of unacceptable material damage at critical high-stress locations
- monitoring of evolution of material damage into critical size
- growth prediction by probabilistic fatigue damage procedure
- adjustments for actual damage state at prescribed intervals
- probabilistic forecast for near term and of lifetime

Schematic of Continuous Lifetime Diagnostic System



Stage 1

Fatigue Damage Leading to Macrocrack Initiation

Damage model

Damage parameter

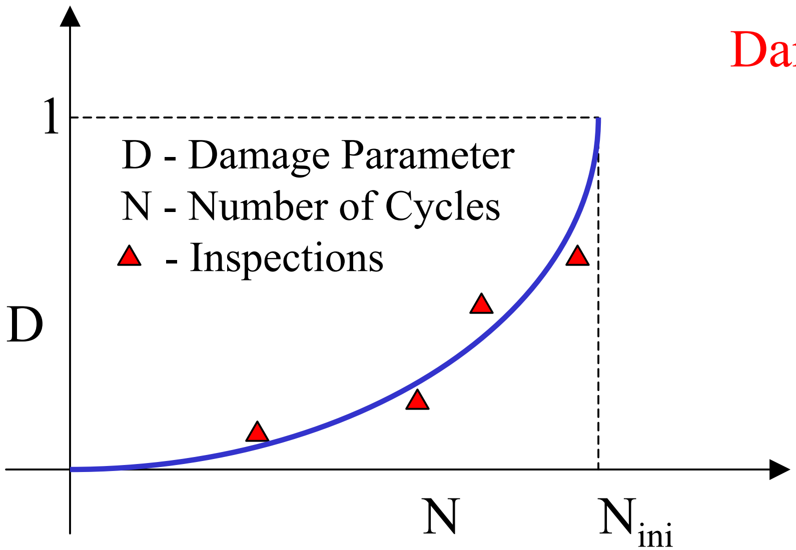
Plan of action

Measurements

Probability of macrocrack initiation

Example

Modeling of Damage in Metals



Damage Model:

$$\frac{dD}{dN} = \frac{1}{N_c}\left\langle 1 - \frac{r_c(\overline{\sigma})}{\Delta\sigma/2}\right\rangle^m \frac{1}{(1-D)^n}$$

$\Delta\sigma$ - Stress range in a cycle

$r_c(\overline{\sigma})$ - endurance limit at $\overline{\sigma}$

N_c - Normalizing constant

Solving for D(N)

$$D(N) = 1 - \left[(1-D_0)^{n+1} - \frac{(N-N_0)}{N_c}\left\langle 1 - \frac{r_c(\overline{\sigma})}{\Delta\sigma/2}\right\rangle^m (n+1)\right]^{\frac{1}{n+1}}$$

D_0 – damage at $N = N_0$ cycles

We choose an equivalent damage parameter, to be measured by structural health monitoring

Measurement of Damage Parameter

- permanently installed ultrasonic sensors
- transmission – reception as fatigue damage progresses
- received pulse is affected by damage
- acoustic nonlinearity: second harmonic amplitude
- changes in attenuations and velocity of signals

Damage Parameter

Acoustic Nonlinearity

Cyclic loading generates various mechanisms on the microscale:

- motion of dislocations
- cracking at grain boundaries
- formation of microcracks

Changes of the microstructure affect the mechanical properties. These can be correlated to the transmission of ultrasound

HARMONIC GENERATION

- generate surface wave at 5 MHz, displacement amplitude A_1
- fatigue mechanisms give rise to a second harmonic at 10 MHz, displacement amplitude A_2

Acoustic nonlinearity parameter (β)

$$\beta = \frac{8A_2}{A_1^2 k^2 l}$$

$k = 2\pi/\text{wavelength}$
$l = \text{distance of travel}$

$$\left.\begin{array}{l}\bullet\ \text{Measure } |A_1| \\ \bullet\ \text{Measure } |A_2|\end{array}\right\}\ \text{at increasing number of cycles}$$

β is a MEASURE OF FATIGUE DAMAGE PRIOR TO ACTUAL MACROCRACKING

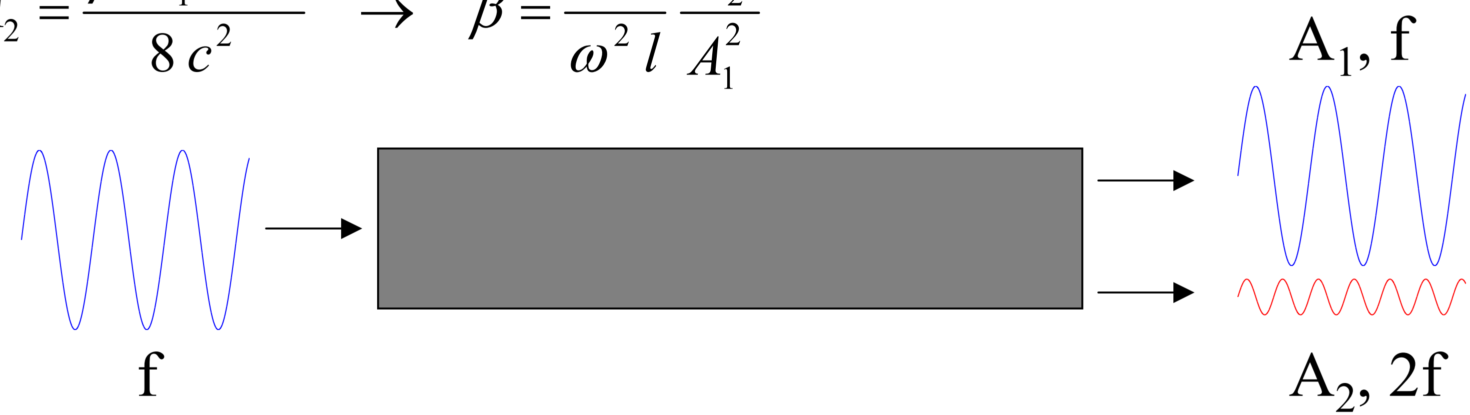
Nonlinear Wave Propagation In a Rod

$$\rho \frac{\partial^2 u}{\partial t^2} = \frac{\partial \sigma}{\partial x} \quad \rightarrow \quad \frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}, \qquad \sigma = E \frac{\partial u}{\partial x}, c^2 = \frac{E}{\rho}$$

$$\frac{\partial^2 u}{\partial t^2} = c^2 \left[1 - \beta \frac{\partial u}{\partial x} \right] \frac{\partial^2 u}{\partial x^2}$$

$$u(l,t) = A_1 \sin(\omega t - kl) - \frac{\beta A_1^2 \omega^2 l}{8 c^2} \cos 2(\omega t - kl) + 😐$$

$$A_2 = \frac{\beta A_1^2 \omega^2 l}{8 c^2} \quad \rightarrow \quad \beta = \frac{8 c^2}{\omega^2 l} \frac{A_2}{A_1^2}$$



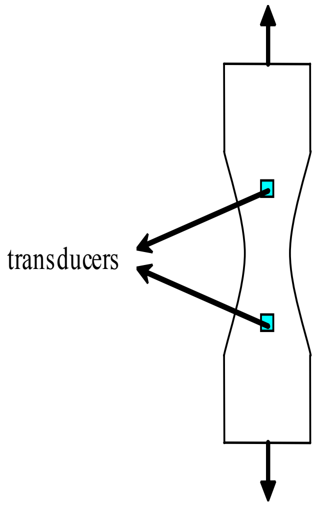
Similar results for surface acoustic waves

Plan of Action

- select a material: 4340 Steel
- set up fatigue test
- instrument the specimen with sensors
- define damage parameter to be measured
- collect sensor data on-line
- verify damage off-line
- define damage evolution functions
- apply probabilistic fatigue procedure
- probabilistic forecast of damage growth
- verify result

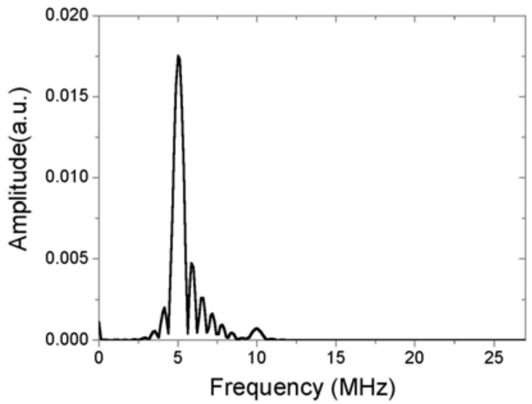
Test Configuration for Feasibility Study

Fatigue Test

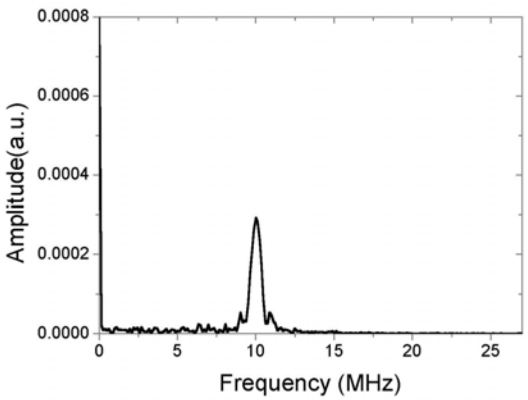


- MTS closed loop electrohydraulic system of 90 kN capacity
- tension-tension, Load controlled: σ_{max} = 950 MPa, σ_{min} = 95 MPa

Fatigued sample
(unfiltered)



Fatigued sample
(filtered)



Probability of Macrocrack Initiation

P_{ma} = probability that the number of cycles to macrocrack initiation will be less than a specified number of cycles N_s

From the damage model:

$$D(N) = 1 - \left[(1 - D_0)^{n+1} - \frac{(N - N_0)}{N_c} \left\langle 1 - \frac{r_c(\overline{\sigma})}{\Delta\sigma / 2} \right\rangle^m (n+1) \right]^{\frac{1}{n+1}}$$

- N = number of cycles
- m and n are deterministic parameters determined from experiments in the lab
- $r_c(\overline{\sigma})$ has a probability density $p_1(r_c(\overline{\sigma}))$ which is taken from literature
- D_0 has a probability density $p_2(D_0)$ which depends on the measurement method

Probability of Macrocrack Initiation

$$P_{ma} = \quad 1 - Pr(D(N_s) < 1)$$

Probability that there is no macrcocrack initiation prior to N_s

where, $$D(N_s) = 1 - \left[(1-D_0)^{n+1} - \frac{(N_s - N_0)}{N_c} \left\langle 1 - \frac{r_c(\overline{\sigma})}{\Delta\sigma/2} \right\rangle^m (n+1) \right]^{\frac{1}{n+1}}$$

Now, $$D(N_s) < 1 \Rightarrow \left[(1-D_0)^{n+1} - \frac{(N_s - N_0)}{N_c} \left\langle 1 - \frac{r_c(\overline{\sigma})}{\Delta\sigma/2} \right\rangle^m (n+1) \right] > 0$$

$$\Rightarrow N_s < N_0 + \frac{N_c}{n+1} (1-D_0)^{n+1} \left\langle \frac{\Delta\sigma/2}{\Delta\sigma/2 - r_c(\overline{\sigma})} \right\rangle^m \equiv N_{ini}$$

Therefore, $D(N_s) < 1 \Rightarrow N_s < N_{ini}$

Hence, $$\begin{aligned} P_{ma} &= \quad 1 - Pr\,(N_s < N_{ini}) \\ &= \quad Pr\,(N_{ini} < N_s) \end{aligned}$$

Calculate this using Monte
Carlo integration

Probability of Macrocrack Initiation

Let

$\boldsymbol{X} = [X_1 \; X_2]$ where $X_1 = r_c(\sigma)$ and $X_2 = D_0$ (represent uncertain quantities)

$f_{\boldsymbol{X}}(\boldsymbol{x})$ = joint probability distribution of $\boldsymbol{X}$

Define: $g = N_{ini} - N_s$

Then,

$g < 0$: region corresponding to $N_{ini} < N_s$, i.e. macrocrack formation

Probability of Macrocrack Initiation, P_{ma}

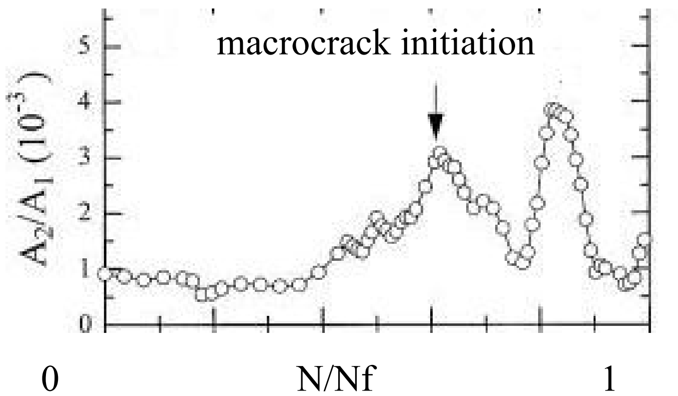
$$P_{ma} = Pr(N_{ini} < N_s) = \int_{g(\boldsymbol{x})<0} f_X(\boldsymbol{x})\, d\boldsymbol{x}$$

Calculate using Monte Carlo Integration with importance sampling

Acoustic Nonlinearity Measurements

Ogi, H., Hirao, M. and Aoki, S. 2001. ``Noncontact monitoring of surface wave nonlinearity for predicting the remaining life of fatigued steels'', J. of App. Phy. 90(1), 438-442.

0.25 % (mass) Carbon steel



j	$Ninsp_j$	$(A_2/A_1)_j$ x 10^{-3}
0	0	0.9
1	5600	0.9
2	11200	0.8
3	16800	0.9
4	22400	0.9
5	26880	1.5
6	30800	2.0
7	33040	2.5
8	34000	3.1

Size of macrocrack at nucleation ~ 0.25 mm (250 microns)

Sample Problem

Aim: To find the probability of macrocrack initiation from the data available in the literature (Ogi, et.al)

To demonstrate the method, these data are used – 1. as laboratory data
2. as inspection data

In practice, data from previous experiments should be available

Sample data for rotating bending fatigue test with four point bending configuration on 0.25% C steel (Ogi, et. al.). Maximum bending stress: 280 MPa, Yield strength: 333 MPa

Measured Values of Acoustic Nonlinearity during Successive Inspections

j	$Ninsp_j$	$(A_2/A_1)_j \times 10^{-3}$
0	0	0.9
1	5600	0.9
2	11200	0.8
3	16800	0.9
4	22400	0.9
5	26880	1.5
6	30800	2.0
7	33040	2.5
8	34000	3.1

normalize →

Damage values during Successive Inspections

j	$Ninsp_j$	'Measured' $Dinsp_j$	Corrected $Dinsp_j$
0	0	0.2769	0.2769
1	5600	0.2769	0.2769
2	11200	0.2462	0.2769
3	16800	0.2769	0.2769
4	22400	0.2769	0.2769
5	26880	0.4150	0.4150
6	30800	0.6150	0.6150
7	33040	0.7692	0.7692
8	34000	0.9539	0.9539

Sample Problem: contd.

- Calculate the constants m and n by using nonlinear regression on the prior data set
- Observing that the damage remains constant up to cycle number 22400, the probability of macrocrack initiation is calculated for cycle number 26880 onwards
- Parameters used in the probability of macrocrack initiation
  - Fixed parameters
    - $\Delta\sigma = 2 \times 280$ MPa
    - $N_c = 10000$
    - m and n determined from laboratory
  - Random (uncertain) parameters
    - $r_c(0)$: Lognormal distribution with mean – 180 MPa and standard deviation – 5.4 MPa
    - D_0: Truncated normal distribution ($0 \leq D_0 \leq 1$) with the mean of the parent normal distribution equal to the observed value of damage at the latest inspection and standard deviation equal to 0.1

Sample Problem: Results

Calculation of probability of macrocrack initiation, P_{ma}

- use Monte Carlo integration to get an estimate of the probability of failure

Calculation of P_{ma}

Cycles	P_{ma}							
N_s	1st Insp (5600)	2nd Insp (11200)	3rd Insp (16800)	4th Insp (22400)	5th Insp (26880)	6th Insp (30800)	7th Insp (33040)	8th Insp (34000)
5600	0.0000							
11200	0.0000	0.0000						
16800		0.0000	0.0000					
22400			0.0003	0.0000				
26880				0.0035	0.0000			
30800					0.1233	0.0000		
33040					0.4384	0.2677	0.0000	
34000					0.5840	0.4894	0.4302	0.0000
35000					0.7133	0.6904	0.7576	0.9373
40000					0.9788	0.9919	0.9991	0.9999

Stages 2 and 3

Growth of a Macrocrack

Crack growth law

Paris' Law

Monitoring of crack growth

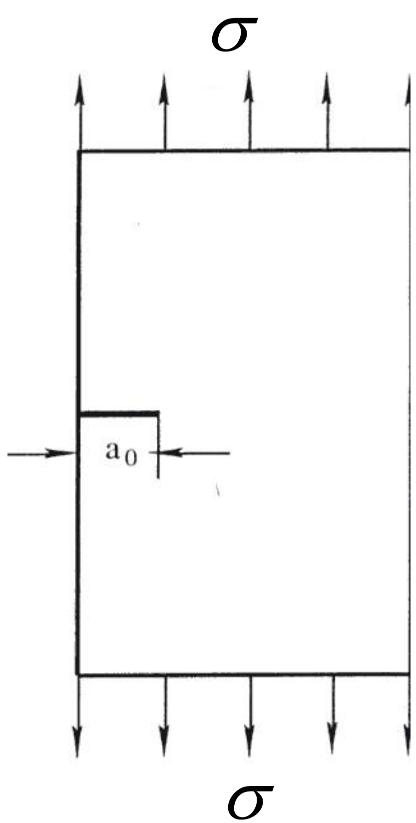
Probability of detection

Example

Probability of undetected $a > a_{cr}$

Example

Edge Crack with Random Initial Length under Tensile Loading



Paris Law

$$\frac{da}{dN} = D(\Delta K)^m$$

N = number of cycles

da/dN = rate of crack growth

D, m = material parameters

ΔK = amplitude of stress intensity factor

$$\Delta K = 1.12\sigma\sqrt{\pi a}$$

$$a_N^{1-m/2} = a_0^{1-m/2} + ND(1-m/2)(1.12\sigma\sqrt{\pi})^m \quad (m \neq 2)$$

Example: Paris Law

$$\frac{da}{dN} = D\left(1.12\,\sigma\,\sqrt{\pi\,a}\right)^m$$

$m = 3.0$

$D = 2.5\times 10^{-11}$

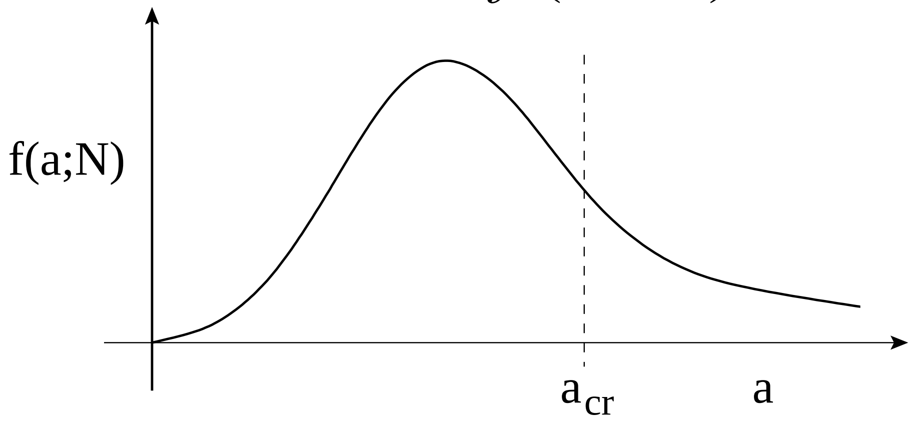
a_0 : lognormal distribution with mean 0.250 mm and standard deviation 0.1 mm

$\sigma = 280$ MPa

$R = -1$

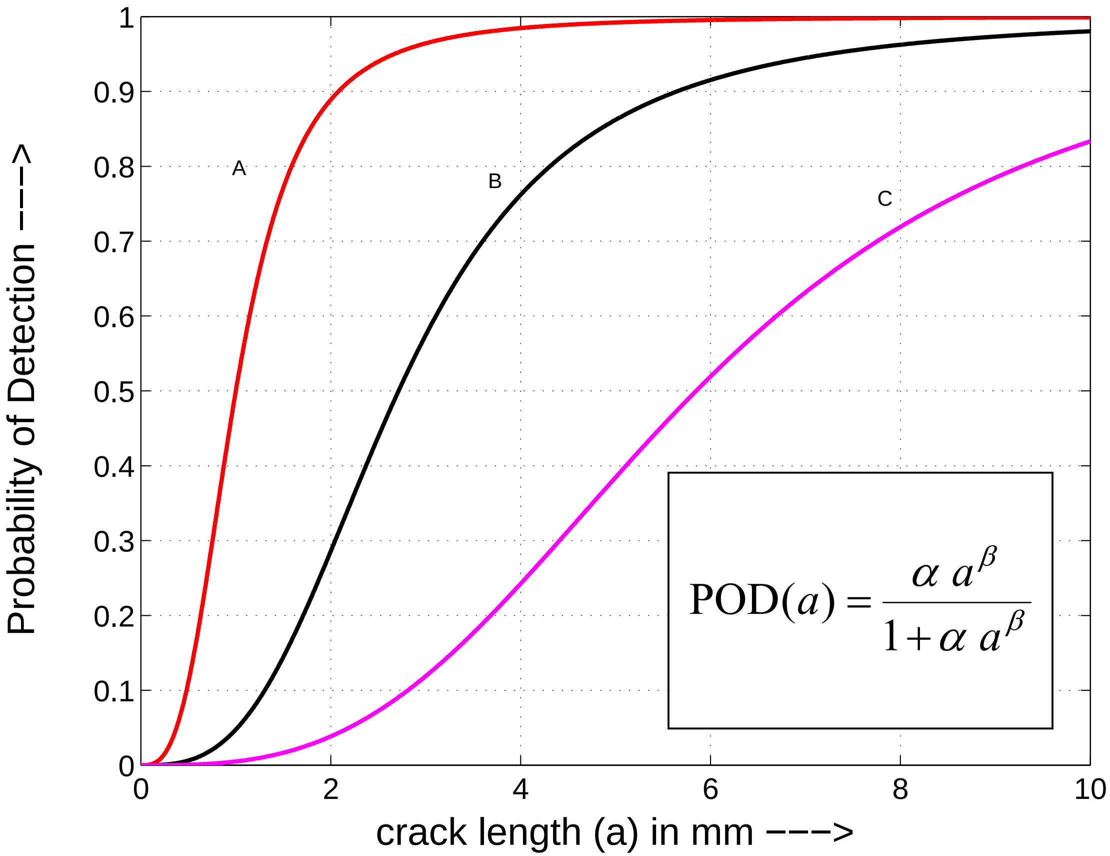
Determine the probability that at a given cycle number N, $a > a_{cr}$

From Paris Law find $f(a\,;N)$



$$\Pr(a > a_{cr}) = \int_{a_{cr}}^{\infty} f(a\,;N)\,da$$

Probability of Detection Curves



$$\mathrm{POD}(a) = \frac{\alpha\, a^{\beta}}{1+\alpha\, a^{\beta}}$$

$$PND(a) = 1 - \frac{\alpha\, a^{\beta}}{1+\alpha\, a^{\beta}} = \frac{1}{1+\alpha\, a^{\beta}}$$

$f(a\,;N)$ = probabilit y density function of the crack length at cycle N

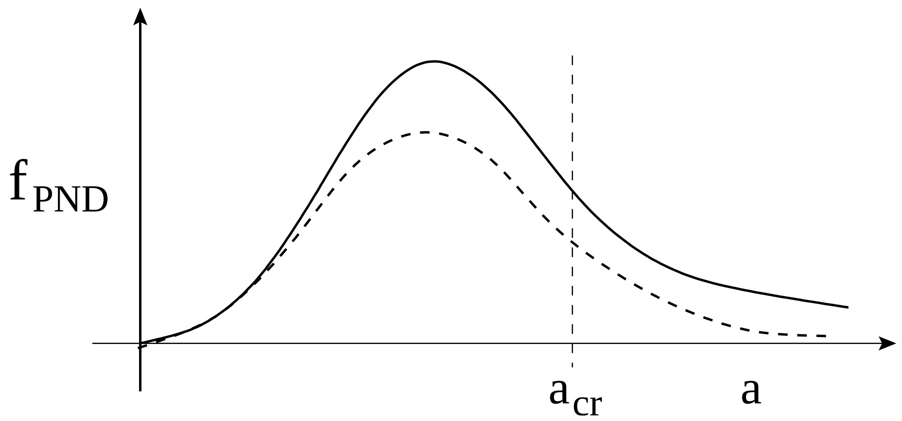
Consider $f(a\,;N)\,PND(a\,;N_i)$

$$\mathrm{A}:\alpha = 1.00\ \mathrm{mm}^{-\beta},\ \beta = 3.0,\ \mathrm{B}:\alpha = 0.05\ \mathrm{mm}^{-\beta},\ \beta = 3.0$$

$$\mathrm{C}:\alpha = 0.005\ \mathrm{mm}^{-\beta},\ \beta = 3.0$$

Effect of PND

Consider $f_{PND} = f(a;N)PND(a;N_i)$



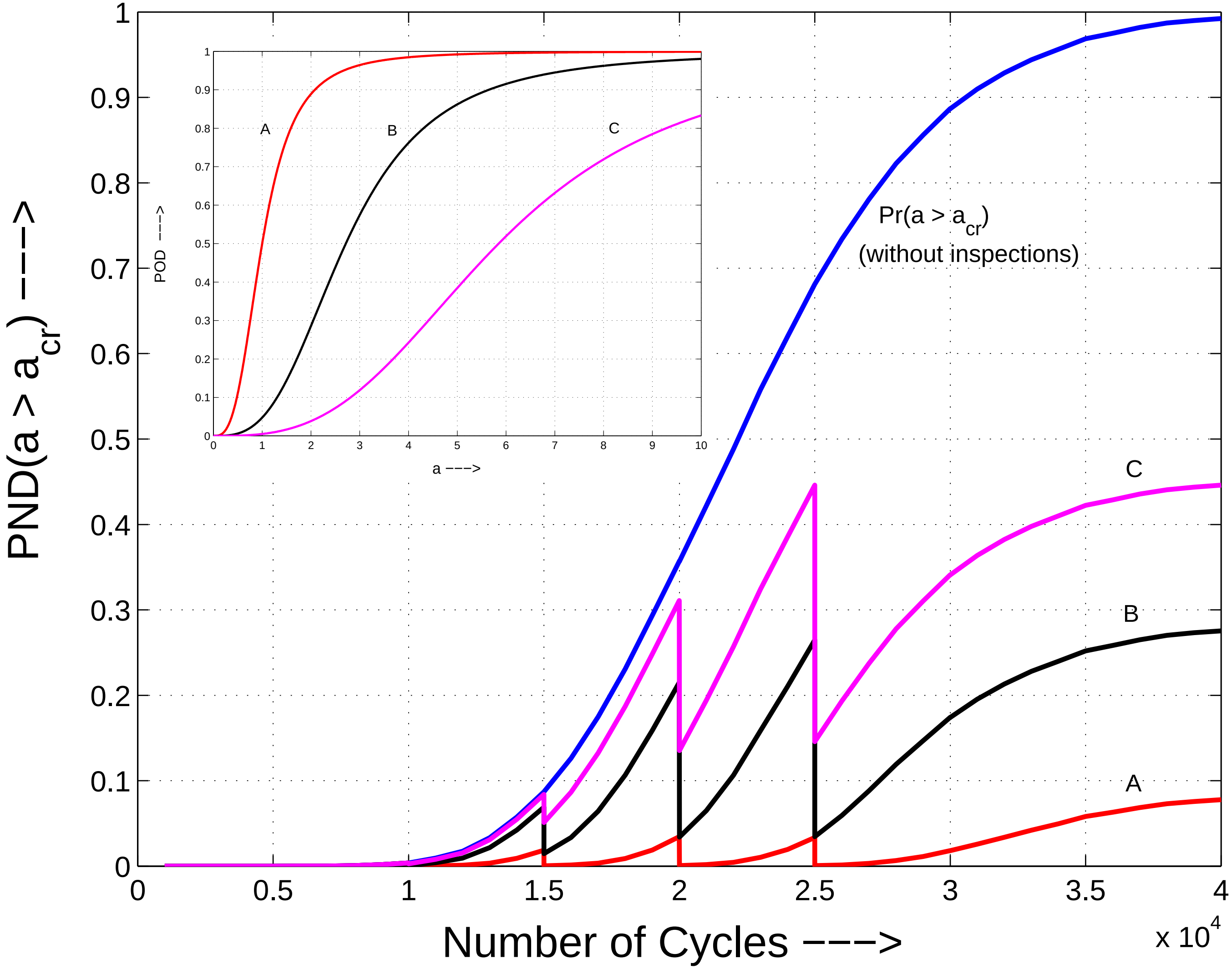
$$PND(a > a_{cr}) = \int_{a_{cr}}^{\infty} f(a;N)PND(a;N_i)\, da$$

Now suppose we have inspections at $N = N_i$, (i = 1…I)

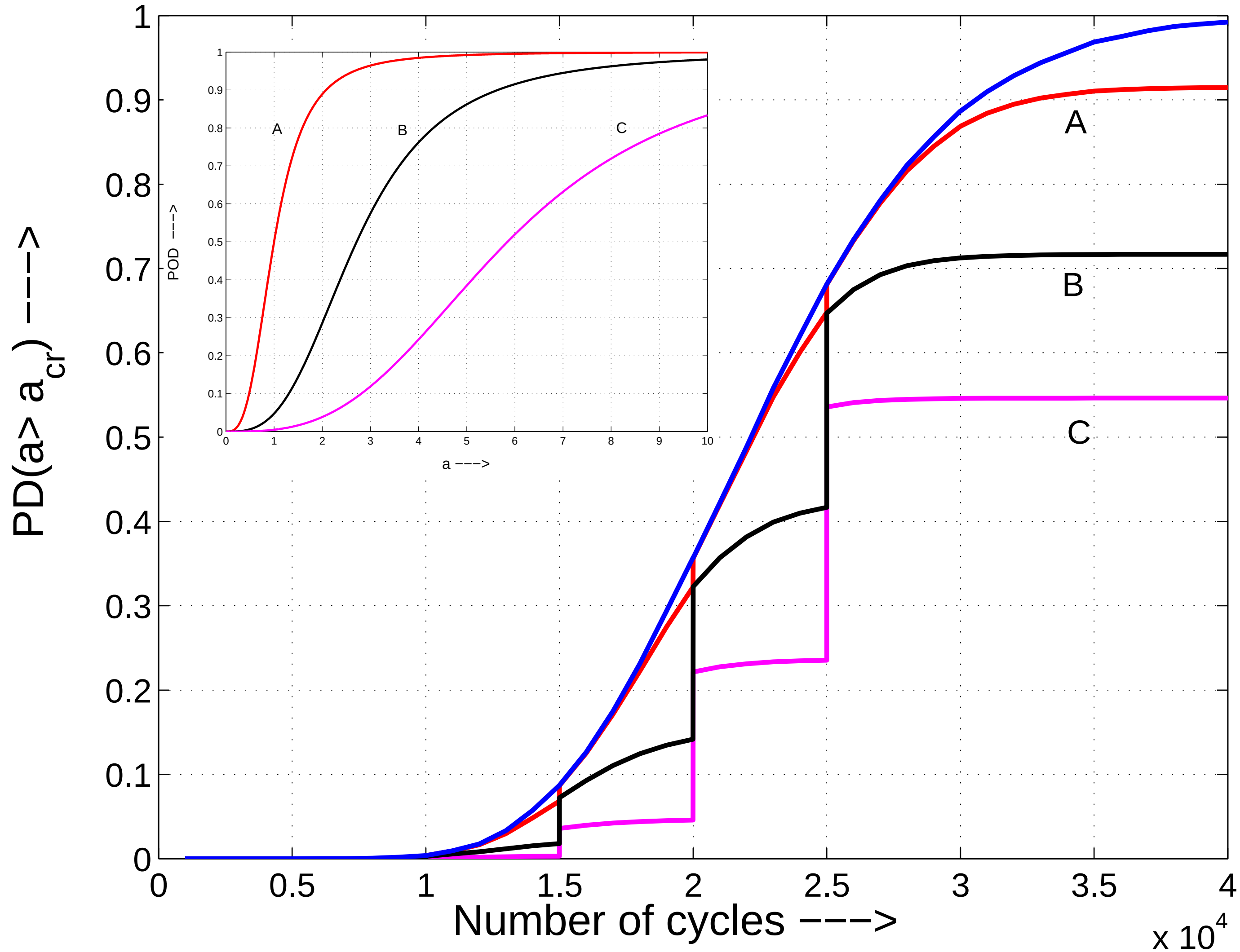
Then, $$f_{PND} = f(a;N)q(a;N_1 \cdots N_I)$$

$$q(a;N_1 \cdots N_I) = \prod_{i=1}^{I} PND(a;N_i)$$

$$PND(a > a_{cr}) = \int_{a_{cr}}^{\infty} f(a;N)q(a;N_i)\, da$$



PND(a > a_cr) --->
Number of Cycles --->
x 10^4
Pr(a > a_cr)
(without inspections)
A
B
C
POD --->
a --->


PD(a> a_cr) --->
Number of cycles --->
x 10^4
A
B
C
POD --->
a --->
A
B
C

Conclusions

A Structural Health Monitoring System that includes the following features has been presented:

- ultrasonic nonlinearity as a damage parameter
- a heuristic damage growth law
- material/other parameters treated as random variables
- periodic measurements to assess state of damage and update state of damage
- probability of macrocrack formation
- probability of undetected $a > a_{cr}$

Critical Issues

- microsensors (IDT, piezo):
  - small
  - autonomous (accelerometer, antenna, battery)
  - cheap, maintainable and repairable
  - accurate, known POD
- coupling to structure
- switching system
- wireless transmission to central station
- data management (instantaneous interpretation ?)
- processing for probabilities of macrocrack formation and subsequent crack propagation to failure
- validation
- **next**
  - Relation to load spectrum
  - Low cycle vs high cycle fatigue
- **still later**
  - Installation on rotorcraft components in laboratory settings
  - Transition to rotorcraft